

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS International General Certificate of Secondary Education

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	
CAMBRIDGE INTERI	NATIONAL MATHEMATICS		0607/05
Paper 5 (Core)			May/June 2010
			1 hour
Candidates answer or	the Question Paper.		
Additional Materials:	Graphics Calculator		

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

Do not use staples, paper clips, highlighters, glue or correction fluid.

You may use a pencil for any diagrams or graphs.

Answer all the questions.

You must show all relevant working to gain full marks for correct methods, including sketches.

In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.

At the end of the examination, fasten all your work securely together.

The total number of the marks for this paper is 24.

This document consists of 4 printed pages.



[Turn over

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Answer all questions.

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INVESTIGATION

FERMAT'S LITTLE THEOREM

The division $46 \div 5$ gives 9 with a remainder of 1. A method for finding the remainder is

$$46 \div 5 = 9.2$$

Because $9 \times 5 = 45$, the remainder is 46 - 45 = 1.

The division $921 \div 7$ gives a remainder of 4.

A method for finding the remainder is

$$921 \div 7 = 131.571...$$

Because $131 \times 7 = 917$, the remainder is 921 - 917 = 4.

The division $2^{11} \div 13$ gives a remainder of 7.

A method for finding the remainder is

$$2^{11} \div 13 = 157.5384...$$

Because $157 \times 13 = 2041$, the remainder is $2^{11} - 2041 = 7$.

Note: To calculate the value of 2⁵

either use the appropriate calculator key

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or use $2^5 = 2 \times 2 \times 2 \times 2 \times 2$.

The value of 2^5 is 32.

- 1 Find the remainder in these divisions.
 - (a) $57 \div 6$

(b) $579 \div 13$

(c) $2^5 \div 7$

.....

(d) $2^9 \div 17$

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2 In 1640 the French mathematician Fermat found something interesting about the remainder when dividing by a **prime** number. Some of his results are shown in the table below.

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Prime	Division	Remainder	Division	Remainder	Division	Remainder
3	$2^2 \div 3$					
5	$2^4 \div 5$		$3^4 \div 5$		$4^4 \div 5$	
7	$2^6 \div 7$	1	$3^6 \div 7$		$4^6 \div 7$	
11	$2^{10} \div 11$		$3^{10} \div 11$		$4^{10} \div 11$	1
	$2^{12} \div 13$					

Complete the unshaded boxes in this table. You may use the space below to show any working.

- 3 Use the patterns you have found in your table to complete the following statements.
 - (a) $7^{12} \div$ has a remainder of .
- 4 From the table $2^6 \div 7$ has a remainder of 1. This means that $2^6 - 1$ will divide by 7 exactly. So $2^6 - 1$ has a **prime** factor of 7.
 - (a) Complete the following statements to show why $7^{12} 1$ has a prime factor of 13.

has a remainder of 1.

This means that will divide by exactly.

So $7^{12}-1$ has a **prime** factor of 13.

(b) Write down a prime factor of $3^{16} - 1$.

The investigation continues on the next page.

Examiner's Use

5	Complete the general statement below.		
	a^{p-1} – 1 has a prime factor of		
	This is called Fermat's Little Theorem.		
6	When $p > 25$ and $a = 3$, write down a statement using Fermat's Little Theorem.		
7	Write down a prime factor, other than 3, of 4 194 303. $[2^{22} = 4\ 194\ 304]$		

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